If the universe is at some time dominated by one of these w/ $\rho_{dom} \propto a^{-n}$ then:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} a^{-n} \Rightarrow \frac{da}{dt} = \sqrt{\frac{8\pi G}{3}} a^{1-\frac{n}{2}} \Rightarrow a^{\frac{n}{2}-1} da = \sqrt{\frac{8\pi G}{3}} dt \Rightarrow a(t) = \left(\frac{n}{2}\sqrt{\frac{8\pi G}{3}}\right)^{\frac{2}{n}} t^{\frac{2}{n}}$$

So for $\rho_{dom} =$
$\begin{cases} \rho_M \propto a^{-3} & \Rightarrow a(t) \propto t^{2/3} \\ \rho_R \propto a^{-4} & \Rightarrow a(t) \propto t^{1/2} \\ \rho_C \propto a^{-2} & \Rightarrow a(t) \propto t \\ \rho_V \propto a^{0} & \Rightarrow a(t) \propto e^{Ht} \end{cases}$ ← have to go back and rederive for this one

$$a^{-1} da = \sqrt{\frac{8\pi G}{3}} dt \quad w/ \sqrt{\frac{8\pi G}{3}} = H$$
$$\ln a = \sqrt{\frac{8\pi G}{3}} t$$
$$a = e^{Ht}$$

Remember that these results are based on a dominant contribution (so we take $\rho_{non-dom} \sim 0$). But do we expect dominant contributions? Well since everything scales differently w/ $a(t)$ yes.

- For small $a$ (early times) $\Rightarrow \rho_R$ dominant
- Large $a$ (late times) $\Rightarrow \rho_V$ dominant
- Intermediate $a$ could have $\rho_M$ or $\rho_C$ dominant

Of course we can use observation to help figure out just how much $\rho_R, \rho_M, \rho_C$ we have (more on that next time).

Note: That unless $\rho_V$ dominates at early time (highly unlikely) $a(t\to 0) \to 0$  $\boxed{\text{Big Bang!}}$

# Cosmology II Our Univers

Last time we learned that a FRW universe dominated at early times by anything other than vacuum energy must have begun w/ a Big Bang. This is of course a naked singularity, though it does not violate the CCC since it is not the result of collapse.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$H = \frac{\dot{a}}{a}$ (current value $H_0$ is measurable)

$a(t), \rho(t), k$ — measurable

$\rho_{tot}$ (discuss various types below)

Cosmology is in some sense an observational science as opposed to an experimental one. In fact one may argue that much of GR is the same since we cannot "create" any significant sources, though we can "experiment" by observing test masses. Furthermore, cosmology is the study of the time-varying history of our universe. And unlike the criterion for any good experiment, it won't repeat itself.

So what do we observe? Much of our precise knowledge is the byproduct of astrophysics. In this, the detailed study of stellar models including both nuclear and gravitational effects have given us a pretty clear prediction of how certain stars should behave, e.g. their luminosity, size, emission spectra, etc. Upon observing these "standard candles" any distortion from their predicted features actually gives us information on the nontrivial geometry through which their light is moving, i.e. the shape of our universe.

UV ⟵⟶ IR

**Emission Spectra:** If we predict an emission spectrum | || |||| for a standard candle, and then observe one that is shifted | || |||| we can relate the size of the redshift to the relative sizes of the universe between the time of emission (long ago) and observation (now), i.e. $\frac{w_o}{w_e} = \frac{a_e}{a_o}$

Redshift factor: $Z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{\lambda_o}{\lambda_e} - 1 = \frac{w_e}{w_o} - 1 = \frac{a_o}{a_e} - 1 \Rightarrow a_e = \frac{a_o}{1+Z}$

- We observe $Z > 0$ for SC $\Rightarrow a_o > a_e$, i.e. expansion.

Recall that $H = \frac{\dot{a}}{a} \Rightarrow H_o = \frac{\dot{a}_o}{a_o}$   $f(t_e \approx t_o) \simeq f(t_o) + (t - t_o) f'(t_o)$

[low redshift]

$a_e \simeq a_o + (t_e - t_o) \dot{a}_o$

$\frac{a_e}{a_o} \simeq 1 + (t_e - t_o) H_o$

Then $Z \simeq \frac{1}{1 + (t_e - t_o) H_o} - 1 \simeq (t_o - t_e) H_o$         $\frac{1}{1-X} \sim 1 + X$

we would like to know this

Assuming the light travelled at $c = 1 \Rightarrow t_o - t_e = d \Rightarrow Z = d H_o$

we can measure this

can we independently measure this?

The notion of "distance" in cosmology is quite complicated. I can think of at least five distinct notions of distance that are useful.
- coordinate distance - this one is useful in computation but not "physical"
- equal time distance to a distant object (how far is it from us now?) Less directly tied to observation.
- observed distance to a distant object (which indicates its distance at the time of light emission that is observed now)
- angular separation - the distance between two distant sources (at approximately the same distance from us)
- angular size of a source — the length across a distant source (note that in contrast to angular separation, this does not expand with time since the object is bound by larger forces)

All of these can be related when we know the full geometry. Fortunately, at small redshift $z$ they all are approximately the same.

Yet another distance that is useful for standard candles is:

measured flux

Luminosity distance : $L = 4\pi d_L^2 F$

"known" luminosity of SC

Knowing $L$ and measuring $F$ let's us calculate $d_L = \sqrt{\frac{L}{4\pi F}}$

For low redshift observations we can use $d = d_L$ and combined w/ $z$ measurements

determine a value for $H_0$ :

$H_0 = 70 \pm 10 \frac{KM}{S Mpc}$ $\Rightarrow$ $d_H = \frac{c}{H_0} = 4.55$ million parsecs   parsec $= 3.086 \times 10^{13}$ km

$t_H = \frac{1}{H_0} = 14.4$ billion yrs

For large redshifts things get much more complicated !

Turning to the other quantities to be determined, define: $\Omega_i \equiv \dfrac{8\pi G}{3H^2}\rho_i = \dfrac{\rho_i}{\rho_{crit}}$ {M, R, C, V

where $\rho_{crit} = \dfrac{3H^2}{8\pi G}$ is the total energy density needed so that $K=0$, i.e. the universe is spatially flat.

If $\sum_i \Omega_i = 1 \quad\Longleftarrow$

then $K = 0$.

Of course we make measurements at the current time so $\Omega_{io} = \dfrac{8\pi G}{3H_o^2}\rho_{io}$

$\Omega_{Mo}$: Look at a cluster and use local gravitational effects to infer mass. Then use density of clusters to extrapolate to large scale.

$\Omega_{Mo} \simeq 0.3 \pm 0.1$

$\Omega_{Ro}$: Cosmic Microwave Background (CMB) arises from relic photons after last scattering (once $e^-, p^+$ cooled to form neutral atoms). We measure a thermal distribution w/ $T = 2.73\,°K$ and hence.

$\Omega_{Ro} \simeq 10^{-4}$

Recall: $\rho_M \propto a^{-3}$, $\rho_R \propto a^{-4}$ so $\rho_{Mo} \gg \rho_{Ro}$ makes sense

$\Omega_{Co}$: From our understanding of the CMB, we can predict anisotropies over a length scale $S$. We then observe the angular size of the anisotropies and comparing to $S$ we can determine the spatial curvature.

$K = +1$ $\qquad\qquad K = 0$ $\qquad\qquad K = -1$



$\theta_o \qquad > \qquad \theta_o \qquad > \qquad \theta_o$
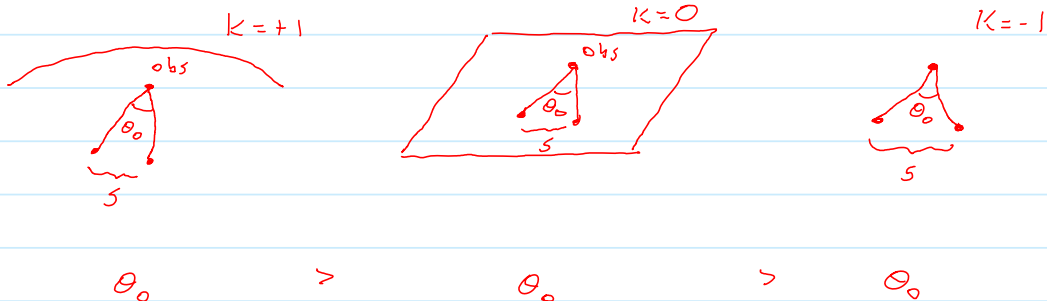
Our observations indicate that $K = 0 \Rightarrow \Omega_{Co} = 0$ and $\Omega_{tot} = 1$

$\Omega_{Vo}$: $\Omega_{tot} = \Omega_{Mo} + \overset{\sim 0}{\cancel{\Omega_{Ro}}} + \overset{0}{\cancel{\Omega_{Co}}} + \Omega_{Vo} = 1 \Rightarrow \Omega_{Vo} \simeq 0.7 \pm 0.1$

## Puzzles in Cosmology

Dark Matter: The result $\Omega_{m0} = 0.3$ is determined from observed gravitational dynamics (by measuring rotational velocities of spiraling galaxies and infering the mass needed to hold them together).
However we can also take note of how much matter we "see", i.e. how much luminous matter there is. That amount, $\Omega_{b0} \simeq 0.04 \pm 0.02$ is largely due to baryonic matter (protons, neutrons).
The balance of $\Omega_{m0}$, i.e. $\Omega_{d0} \simeq 0.26$
- must be non-luminous (hence "dark")
- most likely not M.A.C.H.O.s (massive compact halo objects, e.g. BHs, white dwarfs, neutron stars, etc.)
- most likely non-baryonic (the program of Big Bang Nucleosynthesis is remarkably good at predicting the relative abundances seen today, but also easy to screw up by modifying its assumptions)
- most likely "cold" (since hot would shear out galaxies)
- Axions? (CP-violating $\theta$ parameter in SM is $\sim 0$, to explain PQ proposed making it a field w/
- sterile neutrinos? not part of the weak interactions
- W.I.M.P.s? possibly superpartners, KK modes

but this gives particles!

**Dark Energy:**    Dark energy was coined (by analogy w/ DM) to exhibit our lack of complete understanding of what makes up $\Omega_{vo} \simeq 0.7$.

We expect (and can estimate) a contribution from the zero-point energies of quantum fields in the SM ($\infty$-array of SHOs w/ $\frac{1}{2}\hbar\omega$ ground state).

This contribution would act much like a cosmological constant term $\Lambda g_{\mu\nu}$ in Einstein's eqns. More on this in a minute.

But then we have the "coincidence problem" which is that $\Omega_m$ dilutes w/ $a^{-3}$, while $\Omega_V$ is constant, $a^0$. However we observe $\Omega_{mo} \sim \Omega_{vo}$. Are we just lucky? Enter Mr. Anthropic?!

Actually one method to address this is to introduce a scalar field $\phi$ into EE with a potential $V(\phi)$ that is shallow:
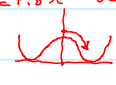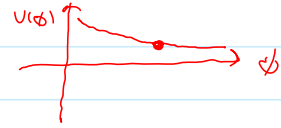


The Friedmann equation becomes:

$$\ddot{\phi} + \underbrace{3H\dot{\phi}} + \frac{dV}{d\phi} = 0$$

acts like "friction" dampening the evolution of $\phi$

Combined w/ a shallow enough potential ($\frac{dV}{d\phi}$ small), we can have $V(\phi)$ be approximately constant but slowly adjusting. However in this case $T_{\mu\nu}$ includes $\underbrace{- V(\phi) g_{\mu\nu}}$

just like $\Lambda g_{\mu\nu}$!

Cosmological Constant Problem: With regards to the value of $\Lambda$, we can actually get an estimate on its _expected_ value by considering various contributions from particle physics. These come in 2 types.

- Anytime a symmetry is spontaneously broken by a field taking a non-symmetric expectation value driven by some effective potential, e.g. ⌣⌣⌣ $]\lambda$, the height of the potential represents the energy released and contributes to $\Lambda$. We know that $q\bar{q}$ condensation which breaks chiral symmetry contributes $10^{34} \text{ eV}^4$ in energy density (here we have taken $c = 1 = \hbar$ so $[x] = [t] = [E]^{-1}$), while the electroweak symmetry breaking phase transition due to the Higg's contributes $10^{44} \text{ eV}^4$.

- We also get a contribution by adding the zero-point energies of the Standard Model fields. Roughly, a field is quantized as an array of harmonic oscillators of varying frequencies over each point of space (the frequency matching the field momentum). But we know that a quantum oscillator has a nonzero ground state of energy $\frac{1}{2}\hbar\omega$. Adding all of these we obviously get $\infty$, but (as usual in particle physics) we expect something else to take over at large enough scales (beyond $m_p$), so we cutoff the momentum integrals at $m_p$, leaving a contribution to $\Lambda$ of order $10^{108} \text{ eV}^4$.

What is observed? $\Omega_{v0} = 0.7 \Rightarrow 10^{-12} \text{ eV}^4$   120 orders of magnitude off!!

Resolving this is one of the more crucial fundamental problems facing particle physics/cosmology.

**Baryon Asymmetry:** One immediately puzzling feature of our universe is that it is comprised almost exclusively of matter w/ almost no anti-matter. This, despite that the Standard Model seems to favor nearly equal production of each. Perhaps surprising is that even though this seems like a maximally one sided distribution today, if we trace it back to early times in the universe, it actual only amounts to a difference in matter and anti-matter of one part in a billion. That means that early on the universe was composed of a lot of matter <u>and</u> anti-matter which (as it cooled) annihilated away and what we see now (all matter) is that tiny initial difference.

Believe it or not, getting a small asymmetry is even harder then getting none or a maximal one (which can be achieved for example by imposing certain symmetries).

This is still an open question, but Sakharov at least enunciated three essential conditions for generating an asymmetry if the universe was born w/ equal amounts of matter/anti.

- B-violating processes (changes net number of baryons)
  However a process that increases $B$ could also increase $\overline{B}$.
- CP-violating process that favors $B$ production over $\overline{B}$.
  However thermal equilibrium would still drive $n_B = n_{\overline{B}}$ even if the rates are different.
- Departure from thermal equilibrium

3 additional puzzles have been more or less addressed by the inflationary models first proposed by Guth.

Flatness problem: $\Omega = 1$ (or $k = 0$) is actually an unstable fixed point under the growth of $a$. We currently see $\Omega = 1$, so are we just lucky? Of course we could just set $\Omega = 1$ and it will stay there, but w/out motivation for doing so this amounts to a fine tuning.

Horizon problem: In a universe of finite age, there can exist regions (at certain times) which have never been in causal contact, i.e. they are separated by a cosmological horizon. At the time the CMB was formed, the universe was large enough to have many causally disconnected regions. Yet the CMB is observed to be remarkably uniform! [Compare w/ ferromagnets]

Relic (or monopole) problem: Whenever a gauge symmetry w/ a $U(1)$ subfactor is spontaneously broken, we would expect the production of at least one topological defect, i.e. magnetic monopole, per causally connected domain. But we observe no monopoles (or vanishingly tiny density).

All of these can be addressed by supposing that the universe, in its past, underwent some period of exponential expansion $a \propto e^{\#t}$ (as opposed to power law $a \propto t^n$). Then:
- During inflationary growth $\Omega = 1$ actually becomes a <u>stable</u> fixed point and so inflation does the "fine-tuning" for us.
- If inflation occurred <u>before</u> the CMB was formed then everything could have been causally connected prior and now only appears to be (based on power law growth).
- If inflation occurred <u>after</u> monopoles were formed, then their density could be "deflated" $\Rightarrow 0$.